Addendum to PRS10 Manual for Firmware Rev. 3.00

Locking to External 1pps

The PRS10 may be locked to an external 1pps source (from a GPS or LORAN receiver, for example) by applying a 1pps pulse to the 1pps input (pin 5 on the main connector). A second order digital phase lock loop (PLL) is used to adjust the frequency of the PRS10 to match the frequency of the 1pps source over long time intervals.

The block diagram of this PLL is shown in Figure 3. The "phase detector" is the time-tagging circuit and firmware, which has a gain of K_{det} = 1bit/ns. The loop filter is a digital filter consisting of an optional prefilter and a standard proportional-integral controller (PI controller) with programmable proportional and integral gains. The VCO is the rubidium frequency standard, whose frequency, f, is tuned by the magnetic field via the SF command parameter with a sensitivity for its 1pps output of K_{vco} = 0.001ns / bit-s, or (1 part in 10^{12}) / bit. The response function for each of the elements of the digital PLL is also indicated in the figure in terms of the standard Laplace variable *s*.

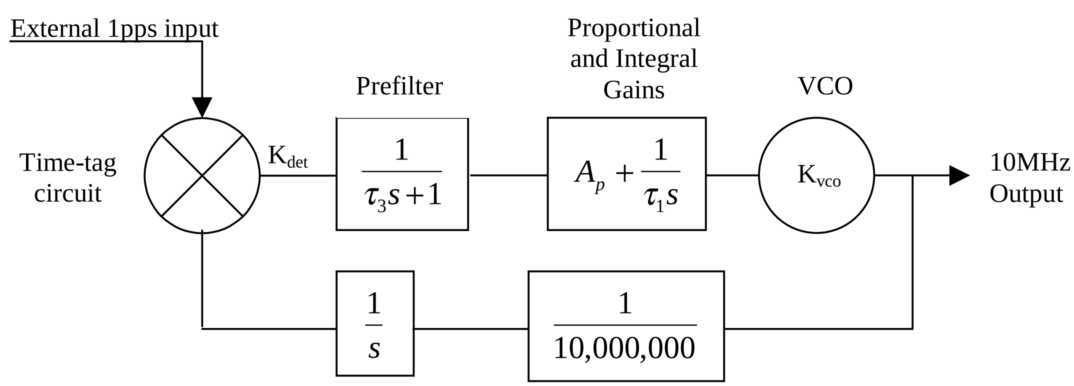

Figure 3. External 1pps Phase Locking Block Diagram

The PI controller is programmed by choosing an appropriate integrator time constant, τ_1, and a stability factor, ζ, which, together with τ_1, determines the proportional gain, A_p, of the controller. τ_1 determines the natural time constant, τ_n, of the PLL for following a step in phase of the reference, while ζ determines the relative rise time and ringing of the PLL in response to the step. The value of ζ also represents the tradeoff in the equivalent noise bandwidth verses peaking in the passband near the natural frequency of the response function.

The PRS10 accepts integrator time constants, τ_1, ranging from 2^8 to 2^{22} seconds in powers of 2. The natural time constant is given by $\tau_n = \sqrt{\tau_1 / K_{\det} K_{vco}} = \sqrt{(1000\text{s})\tau_1}$. Thus, the PRS10 provides natural time constants ranging from 506 seconds to 18.0 hours.

The PRS10 accepts stability factors ranging from 0.25 to 4.0 in powers of 2. The default value of $\zeta = 1.0$ corresponds to a critically damped response; $\zeta < 1.0$ and $\zeta > 1.0$ correspond to underdamped and overdamped responses respectively.

With τ_1 and ζ specified, the proportional gain, A_p, of the controller is given by the equation $A_p = 2\zeta / \sqrt{K_{\det} K_{vco} \tau_1} = 2\zeta / \sqrt{(0.001\text{s}^{-1})\tau_1}$. With the default time constant, τ_1, of 65,536 seconds and a stability factor, ζ, of 1.0, the proportional gain will be about 0.25. In this case the instantaneous frequency of the rubidium source will be adjusted by about 0.25 parts in 10^{12} per nanosecond of time-tag measured.

The PRS10 also provides an optional prefilter. The prefilter is disabled by default, but it can be enabled by sending the command **LM1**, which puts the PRS10 into lock mode 1. If the prefilter is enabled, the PRS10 will exponentially average the time tags output by the "phase detector" before passing the result to the PI controller. The time constant of the prefilter, τ_3, is hard coded to be $\tau_n/6.0$ in order to obtain the maximum benefits of the averaging while simultaneously insuring that the PLL will be stable. The prefilter is disabled by sending the command **LM0**, which puts the PRS10 into lock mode 0, its default mode.

Use of the prefilter is recommended when locking to references that have poorer short term stability than the PRS10, but better long term stability. Locking to the 1pps output by GPS is a prime example of such a case. Use of the prefilter dramatically reduces the digital PLL's sensitivity to the sort term jitter of 50 to 300ns present on the GPS reference 1pps. The GPS reference also has a significant amount of 1/f noise associated with it. Very long time constants are therefore required to prevent the PRS10 from following this noise too closely. The PRS10 provides natural time constants of up to 18.0 hours, which will allow the PRS10 to follow GPS over time scales on the order of a day, but retain the superior short term stability of the rubidium clock. When locking to a reference that has short term stability comparable to the PRS10, disabling the prefilter is recommended because it will allow the PRS10 to better track the phase of the reference.

In lock mode 0, the PRS10's digital PLL will approximate one of the following three equations depending on the value of ζ:

$$\Delta T(t) = \frac{F_0 - \zeta \Delta T(0) / \tau_n}{\sqrt{1-\zeta^2} / \tau_n} e^{\frac{-\zeta t}{\tau_n}} \sin(\sqrt{1-\zeta^2}\, t / \tau_n) + \Delta T(0) e^{\frac{-\zeta t}{\tau_n}} \cos(\sqrt{1-\zeta^2}\, t / \tau_n) \qquad \text{for } \zeta < 1$$

$$\Delta T(t) = t[F_0 - \Delta T(0)/\tau_n]e^{\frac{-t}{\tau_n}} + \Delta T(0)e^{\frac{-t}{\tau_n}} \qquad \text{for } \zeta = 1$$

$$\Delta T(t) = \frac{-\left[F_0 - (\zeta + \sqrt{\zeta^2 - 1})\Delta T(0)/\tau_n\right]}{2\sqrt{\zeta^2 - 1}/\tau_n} e^{\frac{-(\zeta + \sqrt{\zeta^2-1})t}{\tau_n}} + \frac{\left[F_0 - (\zeta - \sqrt{\zeta^2 - 1})\Delta T(0)/\tau_n\right]}{2\sqrt{\zeta^2 - 1}/\tau_n} e^{\frac{-(\zeta - \sqrt{\zeta^2-1})t}{\tau_n}} \qquad \text{for } \zeta > 1$$

ΔT(0) is the initial offset in phase of the PRS10 from the reference. F_0 is the initial offset in frequency of the PRS10 from the reference. *ΔT(t)* details how the PRS10 approaches the phase of the reference as a function of time. With the default time constant, τ_1 = 65,536s, and stability factor, $\zeta = 1$, the PRS10's 1pps output will exponentially approach the phase of the reference 1pps input with a time constant τ_n = 8,095 seconds or approximately 2¼ hours. In lock mode 1, the equations describing *ΔT(t)* are qualitatively similar to those presented above, but generally can only be solved numerically.

The locking algorithm of the PRS10 proceeds as follows:

• The 1pps PLL is enabled when the unit is turned-on or restarted if the PL parameter stored in the unit's EEPROM is "1".

• The PLL will begin to control the frequency of the rubidium frequency standard when 256 consecutive "good" 1pps inputs (i.e., 1pps inputs which are within ±2048ns of the first time-tag result, modulo 1s) are received.

• After receiving 256 consecutive "good" 1pps inputs, the 1pps pulse delay is set to the last of the 256 time-tag values. (For example, if the last of the 256 "good" time tag values is 123,456,789ns then the program will set the 1pps output delay to 123,456,789ns, which moves the 1pps output by 123,456,789ns, so that new time-tag values will be about zero.) Also, the current value of the SF parameter (which adjusts the frequency of the rubidium frequency standard over the range of ± 2000 parts in 10^{12}) is used to initialize the integrator, Int(0). (The current value of the SF parameter may be from the internal calibration pot position, an external calibration voltage, the value from a previously received SF command, or the value left over from a previous PLL lock.) If the prefilter is enabled, the exponential filter for the time tags is zeroed.

• The unit will lock the frequency of the PRS10 to the "good" 1pps input pulses. "Bad" 1pps inputs (1pps inputs with time-tags greater than 1,024ns from the last "good" 1pps input) will be rejected. The frequency parameter, f, to the SF command will be updated with each "good" time-tag result, ΔT(n), as follows:

The prefilter : if **LM0** $\overline{\Delta T}(n+1) = \Delta T(n)$

The prefilter : if **LM1** $\overline{\Delta T}(n+1) = (1.0 - \Delta t/\tau_3)\overline{\Delta T}(n) + (\Delta t / \tau_3)\,\Delta T(n)$

The integral term: $Int(n+1) = Int(n) - (\overline{\Delta T}(n+1) / \tau_1)K_{det}\Delta t$

The proportional term: $Pro(n+1) = -A_p\overline{\Delta T}(n+1)K_{det}$

The frequency setting: $f(n+1) = Pro(n+1) + Int(n+1)$

In the above equations, Δt is the time between phase comparisons, which is one second for the PRS10. The frequency control value, f, ranges over ±2000 bits. If the new f value exceeds 2000, it is set to 2000. If the new f value is less than -2000, it is set to -2000.

If the new integral term exceeds 2000, it is set to 2000. If the new integral term is less than -2000, it is set to -2000. This will prevent "integrator wind-up" in the case that the f-value is pinned for a long time to slew the 1pps output in line with the 1pps input.

The output of the digital filter, f, is used as the frequency control parameter for the SF (set frequency) command, which is updated once a second.

• The PLL will be aborted and restarted if there are 256 consecutive "bad" 1pps inputs. (This could happen if the 1pps input is moved suddenly by more than 1,024ns.) The PLL will also be aborted and restarted if the measured time-tag value for a "good" 1pps input exceeds ±4ns/s * τ_1. (For τ_1's default value of 65,536 seconds, the PLL will restart if the absolute value of a "good" time-tag is greater than 262,144ns. This could happen if the 1pps input is more than a few parts in 10^{-9} off the correct frequency for a long time.)